Rutgers, The State University of New Jersey

15:254:540:01 Understanding School-Aged Students’ Mathematical Learning

3 credits

Fall 2021

Online

Graduate School of Education (CANVAS)

Instructor: Dr. Carolyn A. Maher  carolyn.maher@gse.rutgers.edu
Research Consultant: Dr. Victoria Krupnik  vkrupnik@rutgers.edu

Maher Phone Number 908-400-9793 (cell)  10 Seminary Place, Room 231

Office Hours: Monday & Tuesday  6:30-8:30 PM (by appointment)

Prerequisites or other limitations: none

Mode of Instruction:

___ Lecture
___ Seminar
___ Hybrid
X Online
___ Other

Permission required:

X No
Yes

Non-matriculated students may get a permission number from Jennifer Manson

(jennifer.manson@gse.rutgers.edu)

Faculty Syllabus Statement for Disability Services:

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: https://ods.rutgers.edu/students/documentation-guidelines. If the documentation supports your request for reasonable accommodations, your campus’s disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this
process, please complete the registration form (https://webapps.rutgers.edu/student-ods/forms/registration).

Course catalogue description

Understanding School-Aged Students’ Mathematical Learning (formerly Introduction to Mathematics Education) is required of all graduate students in mathematics education. It is designed to introduce students to theoretical perspectives on learning and teaching mathematics and research about student learning in the content domains of algebra, combinatorics, fractions, and probability. Students will study videos from research studies of children’s reasoning in these domains and discuss the relevance of the videos of student learning to their own practice. Students will also engage in problem solving and readings that will provide a background for the videos that are studied. Course activities support making connections to the NCTM Standards through a final project in which each student creates a multimedia artifact called a VMCAanalytic that illustrates the development of reasoning in a learner or learners and identifies the conditions that support growth in reasoning.

Course purposes, context and methods

Students will engage in a variety of activities done asynchronously online through the CANVAS course website. The online version of the course will be accompanied by three ZOOM meetings to introduce students to tasks and tools that are relevant to the problem-solving activities and video study. During the ZOOM meetings, students will work in small groups on the mathematical tasks, provide justifications for their solutions, and reflect on the variety of strategies and heuristics that emerge and then submit convincing solutions to the problems to their Canvas Assignments. Students will study videos from research studies of children’s problem solving of the same or similar tasks. These activities are designed to deepen understanding of the mathematics content by attending to the process by which learners build meaning of mathematical ideas and justifications of solutions. A focus is to gain insight into the development of reasoning of learners, through sense making, justification and argumentation. Conversations that begin online are sometimes revisited and extended as appropriate throughout the course.

The online course work is partitioned into three content domains: algebra, combinatorics, and fractions/rational numbers. The emphasis is on introducing how conceptual understanding of the mathematical ideas and ways of reasoning can be built by learners in these areas. Throughout the course, there is study of assigned video clips and/or video narratives from the three content strands of students engaged in mathematical problem solving. As a prompt, guiding questions will be offered to initiate online discussions. It is expected that students share the initiative to post a response and begin discussions as well as to respond to the ideas of others. The questions are designed to promote reflection and analyses of the problem solving, readings, and video study and to consider the relevance of what is studied to practice.

Introduction to the Video Mosaic Repository, the VMCAanalytic, and the RUanalytic tool.

See the tutorial that introduces the video collection, its resources, and tool.

VMC Overview
Invited speakers

Students are invited to attend online talks given by visiting scholars. These talks will be announced as they are scheduled.

Learning Goals

1. Students will gain introductory knowledge of the field of mathematics education with a focus on learning and teaching mathematics at the elementary and secondary level.
2. Students will learn about mathematical structures underlying strands of problem tasks from longitudinal and cross-sectional studies from algebra, counting and combinatorics, and fractions/rational numbers. The research that is introduced spans 25+ years of work. The collection of over 4500 hours of video data is preserved at the Robert B. Davis Institute for Learning. A subset of the collection can be accessed at the Video Mosaic Collaborative (videomosaic.org), a Rutgers University Repository.
3. Students will be introduced to research about how students engage with open-ended, challenging tasks as they build justifications of their solutions to problems.
4. Students will learn about the variety of forms of students’ mathematical reasoning through studying videos.
5. Students will learn about research on learning and teaching through assigned readings and videos, and consider the relevance of findings to current teaching practices.
6. Students will learn about the richness of students’ use of a variety of representations.
7. Students will engage in reflection and discussion of their own problem solving in conjunction with the problem solving of colleagues and of students featured in videos.
8. Students will learn about the NCTM and Common Core State Standards and learn to recognize enactment of these standards through video study.
9. Students will learn how to use the RUanalytic tool to create a VMCAntalytic (video narrative) that demonstrates an understanding of the growth of student reasoning and the conditions that foster the development of that reasoning.

Texts and videos

All required readings, video clips and VMCAntalytics can be accessed through the CANVAS website. A supplemental reading list is provided. Other readings will be assigned, as judgment suggests, throughout the course.

Attendance - Students are required to engage in full and active participation each week on the online sessions. The sharing and thoughtful critiquing of the ideas of others are valued, as is thoughtful reflection and connections to practice. If special circumstances (religious observance, school open house, illness) require absence, students are responsible to inform the instructor beforehand and to make up all work shortly thereafter. It is suggested that each student identify a partner who can assist when one is unable to engage in a weekly discussion.

Citations and References - In all written work, including the creation of the VMCAntalytic and final reflection paper, clarity, conciseness, and relevance to the topic of discussion are valued.
Online discussions, while informal, need backing for ideas that are posted. Citations and references in VMCAnalytic final project need to follow APA style.

**Grading** - Grades are based on the thoughtful completion of all assignments in a responsible manner. The following distribution of participation will be used as a guide: participation in the online discussions – 40%; written reactions to readings and/or videos – 20%; problem solving – 10%; VMCAnalytic Final Project and feedback to others – 20%; final reflection – 10%.

**Students are encouraged to attend to feedback from VMCAnalytic Review and submit the VMCAnalytic Final Project for publication on the Video Mosaic Repository.**

**Academic Integrity Policy** - Any violation of academic honesty is a serious offense and is therefore subject to an appropriate penalty. Refer to: [http://academicintegrity.rutgers.edu/integrity.shtml](http://academicintegrity.rutgers.edu/integrity.shtml) for a full explanation of policies.

**Course Requirements** - You are expected to be an active participant in the class through online thoughtful contributions and as a responsive member of the class community. Successful completion of the course requires that you engage in all activities, and create a video narrative (VMCAnalytic). A tutorial is provided.

1. Submit all assignments **ON TIME**.
2. Actively participate in online discussions as you engage with assignments (readings and videos) and respond to guiding questions as posted on the CANVAS course web site. You are required to make at least one original posting and respond to at least two group member postings per week.
3. Be knowledgeable of all the assigned readings, problem tasks, videos, and RUanalytic work.
4. Create a VMCAnalytic using the RUanalytic tool from the video collection housed on the Video Mosaic ([videomosaic.org](http://videomosaic.org)) showing growth in student reasoning and the conditions contributing to that growth (This satisfies a portfolio requirement for those students matriculated in the mathematics-education, MEd, program).
5. Complete a **reflection paper** to include your learning through online discussions about videos, problem solving, readings, project work, collaborative work with peers, and your knowledge of the mathematics, research on how students learn, and implications for teaching with regard to NCTM and Common Core Standards. You may review your postings on the course web site and notes from problem solving and sharing of solutions as you develop your reflective assessment.

**Description of Activities** - See Course Outline below for schedule and requirements. Other readings and video/VMCAnalytic viewings will be assigned throughout the course. Modifications in assignments such as readings and video/analytic study will be made as judgment suggests.

Class sessions are held online. **Three additional synchronous, group problem-solving sessions will be held as new tasks are introduced using ZOOM. These special sessions will be announced, as needed, for students who would like to work collaboratively on tasks**
and/or obtain assistance with their course project. Attendance at these sessions, while voluntary, is strongly recommended.


1. Check the discussion frequently and respond appropriately and on the subject.
2. Focus on one subject per message and use pertinent, informative, and not-too-long titles
3. Capitalize words only to highlight a point or for titles; otherwise viewed as SHOUTING.
4. Be professional, respectful, and careful with your online interaction
5. Cite all quotes, references, and sources.
6. When posting a long message, it is generally considered courteous to warn readers at the beginning of the message that it is a lengthy post.
7. It is inappropriate to forward someone else’s message(s) without their permission.
8. Use humor carefully. The absence of face-to-face cues can cause humor to be misinterpreted as criticism or flaming (angry, antagonistic criticism). Feel free to use emoticons such as :-) or ;-), to let others know that you’re being humorous.

COURSE SCHEDULE

The following outline is a tentative course schedule. Please refer to the CANVAS website weekly for exact assignments. From time to time, we may find it helpful to schedule some synchronous virtual interactions either through a conference call or webinar (e.g., ZOOM). As need or interest suggests, these will be scheduled.

<table>
<thead>
<tr>
<th>WEEK</th>
<th>ACTIVITIES</th>
<th>READINGS</th>
</tr>
</thead>
</table>
| Wk 1 | I. Canvas Assignments  
Complete your background information. Submit to Canvas Assignments. Title background BEGINNING with your name.  
II. Download & review: K-8 Algebra Common Core State Standards Initiative (2010). | I. Readings for Week 1  
II. Online discussion per guiding questions about the readings |
| 9/1 – 9/7 | | |
| Wk 2 | I. Canvas Assignments  
Solve Geese, Ladders, and Museum problems and submit to Canvas Assignments; save file beginning with your name.  
II. Study the following VMCAAnalytics  
1. Kayla Albrethsen: James’ Recognition of the Isomorphism Between the Museum Problem and | I. Readings for Week 2:  
| 9/8 – 9/14 | | |
|-------------------|----------------------------------------------------------------------------------|
| http://dx.doi.org/doi:10.7282/t3-b6j1-4r22 | II. Online discussion per guiding questions about the reading |
| 2. Ariel Constructing Linear Equations for "Guess My Rule" and the "Ladder" Problems | III. Discuss what is meant by “algebraic reasoning”. |
| http://dx.doi.org/doi:10.7282/T3NG4SD7 | |
| 3. Tracing Ariel’s Algebraic Problem Solving: A Case Study of Cognitive and Language Growth | |
| https://doi.org/doi:10.7282/T3N0186C | |

<table>
<thead>
<tr>
<th><strong>I. Study the following VMCAAnalytics:</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I. This week is devoted to individual and partner problem solving, in preparation for studying how students reason and build justifications to problem tasks.</td>
<td></td>
</tr>
<tr>
<td>II. Canvas Assignments Solutions</td>
<td></td>
</tr>
<tr>
<td>Submit convincing arguments to justifying the number of:</td>
<td></td>
</tr>
<tr>
<td>(a) Towers, 3-tall, selecting from 3 colors</td>
<td></td>
</tr>
<tr>
<td>(b) Towers, 5-tall, selecting from 2 colors</td>
<td></td>
</tr>
<tr>
<td>I. Readings for Week 3:</td>
<td></td>
</tr>
<tr>
<td>II. Online discussion per guiding questions about the reading</td>
<td></td>
</tr>
<tr>
<td>I. Readings for Week 4</td>
<td></td>
</tr>
<tr>
<td>Combinatorics and Reasoning book (Maher, Powell &amp; Uptegrove, Eds.)</td>
<td></td>
</tr>
<tr>
<td>1. Chapter 1: The Longitudinal Study</td>
<td></td>
</tr>
<tr>
<td>2. Chapter 2: Methodology</td>
<td></td>
</tr>
<tr>
<td>II. Online discussion per guiding questions about readings</td>
<td></td>
</tr>
</tbody>
</table>
### Wk 5 9/29-10/5

**Students’ Problem Solving in the Counting Strand: Examining Students’ Strategies and Solutions**

**I. Study the following VMCAnalytics**

1. Stephanie, Dana, Jeff and Milin: Gang of 4 (Gr 4)
   - [http://dx.doi.org/doi:10.7282/T3CC0ZND](http://dx.doi.org/doi:10.7282/T3CC0ZND)

2. PUP-Math Pizzas (Gr 5) Parts 1 and 2
   - [http://dx.doi.org/doi:10.7282/T3HM57PQ](http://dx.doi.org/doi:10.7282/T3HM57PQ)
   - [http://dx.doi.org/doi:10.7282/T3NC60FW](http://dx.doi.org/doi:10.7282/T3NC60FW)

3. Inductive Reasoning (Gr5)
   - [http://dx.doi.org/doi:10.7282/T39C707G](http://dx.doi.org/doi:10.7282/T39C707G)

**II. Schedule Zoom meeting with partner or small group to share and discuss problem solutions.**

### Wk 6 10/6-10/12

**Students’ Problem Solving in the Counting Strand: Studying students ‘reasoning and use of representations**

**I. Study the following VMCAnalytics:**

1. PUP-Math Brandon Interview
   - [http://dx.doi.org/doi:10.7282/T3VX0FRD](http://dx.doi.org/doi:10.7282/T3VX0FRD)

2. Brandon’s Aha
   - [http://dx.doi.org/doi:10.7282/T3VH5R01](http://dx.doi.org/doi:10.7282/T3VH5R01)

**II. Canvas Assignments Solutions**

### I. Readings for Week 5

Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.)

1. Chapter 4: Towers, Schemes, Strategies, Arguments
2. Chapter 5: Building an Inductive Argument
3. Chapter 6: Making Pizzas: Reasoning by Cases and by Recursion

### II. Online discussion per guiding questions about readings and video narratives
### Wk 7
10/13-10/19

**Students’ Problem Solving in the Counting Strand: Attending to Problem Structure**

**I. Study the following VMCAalytics:**

1. Romina’s Proof to Ankur’s Challenge (Gr. 10)
   
   [http://dx.doi.org/doi:10.7282/T30P0Z85](http://dx.doi.org/doi:10.7282/T30P0Z85)

2. Stephanie’s Algebraic Solution (Gr 8)
   
   [http://dx.doi.org/doi:10.7282/T3FN180C](http://dx.doi.org/doi:10.7282/T3FN180C)

3. Stephanie’s Geometric Reasoning
   
   [http://dx.doi.org/doi:10.7282/T3QZ2CRF](http://dx.doi.org/doi:10.7282/T3QZ2CRF)

**II. Schedule Zoom meeting with partner or small group to share and discuss problem solutions.**

---

### Wk 8
10/20-10/26

**Students’ Problem Solving Counting Strand: Attending to Structure and Building Connections**

**Study the following VMCAalytics:**

1. PUP-Math Night Session
   
   [http://dx.doi.org/doi:10.7282/T34F1Q0W](http://dx.doi.org/doi:10.7282/T34F1Q0W)

2. PUP-Math, Stephanie (Gr 8) Pascal’s Addition
   
   [http://dx.doi.org/doi:10.7282/T3862FPR](http://dx.doi.org/doi:10.7282/T3862FPR)

3. Pascal’s Identity (Grade 11)
   
   [http://dx.doi.org/doi:10.7282/T3VX0FMM](http://dx.doi.org/doi:10.7282/T3VX0FMM)

4. Developing Isomorphic Relationships (Grade 11)
   
   [http://dx.doi.org/doi:10.7282/T3H1310N](http://dx.doi.org/doi:10.7282/T3H1310N)

---

### Wk 9
10/27-11/2

**Fraction Strand**

**I. Studying how to create a VMCAalytic and how to produce a first draft. Review Tutorial:** Making a VMCAalytic (with RUanalytic tool)

---

### I. Readings for Week 7

   
   Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.)

2. Chapter 8: Ankur’s Challenge

**II. Online discussion per guiding questions about readings, video, and analytic**

---

### I. Readings for Week 8


   Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.):

2. Chapter 12: Representations and Standard Notation

**II. Online discussion per guiding questions about readings, video, and analytic**

---

### I. Readings for Week 9:

<table>
<thead>
<tr>
<th>II. Obtaining, as needed assistance in creating VMCAntalytic and learning to use RUanalytic tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>III. Study the following video clip: Introduction to Informal Math Learning Project (IML): Establishing Norms with Sixth Graders via Rods Lesson: <a href="https://drive.google.com/file/d/1ZuMfdYdlJ787JxfkO0H96dkhcnOaPqg/view">https://drive.google.com/file/d/1ZuMfdYdlJ787JxfkO0H96dkhcnOaPqg/view</a></td>
</tr>
<tr>
<td>IV. Synchronous Session, to be scheduled, to discuss rod models and creating VMCAntalytic</td>
</tr>
</tbody>
</table>

| II. Draft of Analytic Outline. Be sure to include the transcript and time codes for the Video Events. |

---

**Children’s Reasoning Fraction Strand**

**I. Study the following VMCAntalytics:**


2. Task Design Prompts Fourth Grade Students [http://dx.doi.org/doi:10.7282/T3ZK5JF0](http://dx.doi.org/doi:10.7282/T3ZK5JF0)

3. Fourth Graders Reason by Cases as They Explore Fraction Ideas [http://dx.doi.org/doi:10.7282/T3Q2420N](http://dx.doi.org/doi:10.7282/T3Q2420N)

4. An Introduction to Comparing Unit Fractions [http://dx.doi.org/doi:10.7282/T3V4010R](http://dx.doi.org/doi:10.7282/T3V4010R)

**II. Online discussion per guiding questions about readings, video, and analytic**

**III. First draft of your VMCAntalytic to be shared with a partner:** Partner with at least one other student to share and receive feedback about your VMCAntalytic; also, partner with at least one other student to provide feedback on their VMCAntalytic. Please note that the file title should begin with your NAME.

---

**I. Readings for Week 10**

1. Chapter 2: Establishing a Mathematical Community, Gerstein

2. Chapter 3: A Problem with No Solution, Yankelewitz & Winter

3. Chapter 5: Reasoning by Cases while Exploring Fractions as Numbers, Winter & Yankelewitz

4. Chapter 7: Establishing the Importance of the Unit, Van Ness & Alston

**II. Online discussion per guiding questions about readings, video, and analytic**

**III. First draft of your VMCAntalytic to be shared with a partner:** Partner with at least one other student to share and receive feedback about your VMCAntalytic; also, partner with at least one other student to provide feedback on their VMCAntalytic. Please note that the file title should begin with your NAME.

---

**I. Readings for Week 11**

---

**Children’s Reasoning Fraction Strand**

**I. Study the Fraction Strand VMCAntalytics:**

---

**II. Online discussion per guiding questions about readings, video, and analytic**

---

**III. First draft of your VMCAntalytic to be shared with a partner:** Partner with at least one other student to share and receive feedback about your VMCAntalytic; also, partner with at least one other student to provide feedback on their VMCAntalytic. Please note that the file title should begin with your NAME.

---

**I. Readings for Week 11**

---

**Children’s Reasoning Fraction Strand**

**I. Study the Fraction Strand VMCAntalytics:**

---

**II. Online discussion per guiding questions about readings, video, and analytic**

---

**III. First draft of your VMCAntalytic to be shared with a partner:** Partner with at least one other student to share and receive feedback about your VMCAntalytic; also, partner with at least one other student to provide feedback on their VMCAntalytic. Please note that the file title should begin with your NAME.
<table>
<thead>
<tr>
<th>Date</th>
<th>Activities</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>II. Online discussion per guiding questions about readings, video, and analytic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III. Feedback to/from partner’s VMCAAnalytic project</td>
</tr>
<tr>
<td>Wk 12</td>
<td></td>
<td>I. Readings for Week 12</td>
</tr>
<tr>
<td>11/23</td>
<td></td>
<td>2. Chapter 17: Comparing and Ordering Fractions, Horwitz &amp; Schmeelk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3. Chapter 18: Extending Fraction Placement from Segments to Number Line, Horwitz</td>
</tr>
<tr>
<td></td>
<td>I. Study the Fraction Strand VMCAAnalytics:</td>
<td>II. Online discussion per guiding questions about readings and analytics</td>
</tr>
<tr>
<td></td>
<td>1. Imagining the Density of Fractions [<a href="http://dx.doi.org/doi:10.7282/T3FJ2JKN">http://dx.doi.org/doi:10.7282/T3FJ2JKN</a>]</td>
<td>III. PROJECT:</td>
</tr>
<tr>
<td></td>
<td>2. Using Meredith’s Models to Reason About Comparing and Ordering Unit Fractions [<a href="http://dx.doi.org/doi:10.7282/T33J3FQG">http://dx.doi.org/doi:10.7282/T33J3FQG</a>]</td>
<td>Review and revise your VMCAAnalytic as appropriate from feedback received</td>
</tr>
<tr>
<td></td>
<td>3. Extending Fraction Placements from Segments to Number Line: Obstacles and Their Resolutions [<a href="http://dx.doi.org/doi:10.7282/T39Z96SR">http://dx.doi.org/doi:10.7282/T39Z96SR</a>]</td>
<td></td>
</tr>
<tr>
<td>Wk 13</td>
<td>Thanksgiving Recess</td>
<td>No class scheduled</td>
</tr>
<tr>
<td>11/24- 11/24-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11/29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
I. Reporting/Reviewing VMCAnalytic Projects in online discussion

II. Begin Reflection paper

Submit VMCAnalytic: Instructors must approve VMCAnalytic text before putting text online.

Canvas Assignments
1. FINAL VMCAnalytic Project due
2. Reflection paper due

Required Text (Available as paper back and online version – rental also available)


Recommended Text (rental also available)


https://www.amazon.com/Combinatorics-Reasoning-Representing-Isomorphisms-Mathematics-ebook-dp-B008BC0LS0/dp/B008BC0LS0/ref=mt_kindle?_encoding=UTF8&me=&qid=

Supplemental Readings


