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Graduate School of Education

Learning and Teaching
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Rutgers, The State University of New Jersey
10 Seminary Place
New Brunswick, NJ 08901-1183

http://gse.rutgers.edu/

Ph: 732-848-0789

Rutgers, The State University of New Jersey

15:254:540:01 Understanding School-Aged Students' Mathematical Learning

3 credits

Fall 2021

Online

Graduate School of Education (CANVAS)

Instructor: Dr. Carolyn A. Maher carolyn.maher@gse.rutgers.edu

Research Consultant: Dr. Victoria Krupnik vkrupnik@rutgers.edu

Maher Phone Number 908-400-9793 (cell) 10 Seminary Place, Room 231

Office Hours: Monday & Tuesday
6:30-8:30 PM (by appointment)

Prerequisites or other limitations: none

Mode of Instruction:

___ Lecture

___ Seminar

___ Hybrid

_X_Online

___ Other

Permission required:

_X_ No

__Yes

Non-matriculated students may get a permission number from Jennifer Manson

(jennifer.manson@gse.rutgers.edu)

Faculty Syllabus Statement for Disability Services:

Rutgers University welcomes students with disabilities into all of the University's educational programs. In order to receive consideration for reasonable accommodations, a student with a disability must contact the appropriate disability services office at the campus where you are officially enrolled, participate in an intake interview, and provide documentation: https://ods.rutgers.edu/students/documentation-guidelines. If the documentation supports your request for reasonable accommodations, your campus's disability services office will provide you with a Letter of Accommodations. Please share this letter with your instructors and discuss the accommodations with them as early in your courses as possible. To begin this

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process, please complete the registration form (https://webapps.rutgers.edu/student-ods/forms/registration).

Course catalogue description

Understanding School-Aged Students' Mathematical Learning (formerly Introduction to Mathematics Education) is required of all graduate students in mathematics education. It is designed to introduce students to theoretical perspectives on learning and teaching mathematics and research about student learning in the content domains of algebra, combinatorics, fractions, and probability. Students will study videos from research studies of children's reasoning in these domains and discuss the relevance of the videos of student learning to their own practice. Students will also engage in problem solving and readings that will provide a background for the videos that are studied. Course activities support making connections to the NCTM Standards through a final project in which each student creates a multimedia artifact called a VMCAnalytic that illustrates the development of reasoning in a learner or learners and identifies the conditions that support growth in reasoning.

Course purposes, context and methods

Students will engage in a variety of activities done asynchronously online through the CANVAS course website. The online version of the course will be accompanied by three ZOOM meetings to introduce students to tasks and tools that are relevant to the problem-solving activities and video study. During the ZOOM meetings, students will work in small groups on the mathematical tasks, provide justifications for their solutions, and reflect on the variety of strategies and heuristics that emerge and then submit convincing solutions to the problems to their Canvas Assignments. Students will study videos from research studies of children's problem solving of the same or similar tasks. These activities are designed to deepen understanding of the mathematics content by attending to the process by which learners build meaning of mathematical ideas and justifications of solutions. A focus is to gain insight into the development of reasoning of learners, through sense making, justification and argumentation. Conversations that begin online are sometimes revisited and extended as appropriate throughout the course.

The online course work is partitioned into three content domains: algebra, combinatorics, and fractions/rational numbers. The emphasis is on introducing how conceptual understanding of the mathematical ideas and ways of reasoning can be built by learners in these areas. Throughout the course, there is study of assigned video clips and/or video narratives from the three content strands of students engaged in mathematical problem solving. As a prompt, guiding questions will be offered to initiate online discussions. It is expected that students share the initiative to post a response and begin discussions as well as to respond to the ideas of others. The questions are designed to promote reflection and analyses of the problem solving, readings, and video study and to consider the relevance of what is studied to practice.

Introduction to the Video Mosaic Repository, the VMCAnalytic, and the RUanalytic tool.

See the tutorial that introduces the video collection, its resources, and tool.

VMC Overview

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Invited speakers

Students are invited to attend online talks given by visiting scholars. These talks will be announced as they are scheduled.

Learning Goals

1. Students will gain introductory knowledge of the field of mathematics education with a focus on learning and teaching mathematics at the elementary and secondary level.
2. Students will learn about mathematical structures underlying strands of problem tasks from longitudinal and cross-sectional studies from algebra, counting and combinatorics, and fractions/rational numbers. The research that is introduced spans 25+ years of work. The collection of over 4500 hours of video data is preserved at the Robert B. Davis Institute for Learning. A subset of the collection can be accessed at the Video Mosaic Collaborative (videomosaic.org), a Rutgers University Repository.
3. Students will be introduced to research about how students engage with open-ended, challenging tasks as they build justifications of their solutions to problems.
4. Students will learn about the variety of forms of students' mathematical reasoning through studying videos.
5. Students will learn about research on learning and teaching through assigned readings and videos, and consider the relevance of findings to current teaching practices.
6. Students will learn about the richness of students' use of a variety of representations.
7. Students will engage in reflection and discussion of their own problem solving in conjunction with the problem solving of colleagues and of students featured in videos.
8. Students will learn about the NCTM and Common Core State Standards and learn to recognize enactment of these standards through video study.
9. Students will learn how to use the RUanalytic tool to create a VMCAnalytic (video narrative) that demonstrates an understanding of the growth of student reasoning and the conditions that foster the development of that reasoning.

Texts and videos

All required readings, video clips and VMCAnalytics can be accessed through the CANVAS website. A supplemental reading list is provided. Other readings will be assigned, as judgment suggests, throughout the course.

Attendance - Students are required to engage in full and active participation each week on the online sessions. The sharing and thoughtful critiquing of the ideas of others are valued, as is thoughtful reflection and connections to practice. If special circumstances (religious observance, school open house, illness) require absence, students are responsible to inform the instructor beforehand and to make up all work shortly thereafter. **It is suggested that each student identify a partner who can assist when one is unable to engage in a weekly discussion.**

Citations and References - In all written work, including the creation of the VMCAnalytic and final reflection paper, clarity, conciseness, and relevance to the topic of discussion are valued.

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Online discussions, while informal, need backing for ideas that are posted. Citations and references in VMCAnalytic final project need to follow APA style.

Grading - Grades are based on the thoughtful completion of all assignments in a responsible manner. The following distribution of participation will be used as a guide: participation in the online discussions –

40%; written reactions to readings and/or videos – 20%; problem solving – 10%;

VMCAnalytic Final Project and feedback to others – 20%; final reflection – 10%.

Students are encouraged to attend to feedback from VMCAnalytic Review and submit the VMCAnalytic Final Project for publication on the Video Mosaic Repository.

Academic Integrity Policy - Any violation of academic honesty is a serious offense and is therefore subject to an appropriate penalty. Refer to: http://academicintegrity.rutgers.edu/integrity.shtml for a full explanation of policies.

Course Requirements - You are expected to be an active participant in the class through online thoughtful contributions and as a responsive member of the class community. Successful completion of the course requires that you engage in all activities, and create a video narrative (VMCAnalytic). A tutorial is provided.

1. Submit all assignments **ON TIME.**
2. Actively participate in online discussions as you engage with assignments (readings and videos) and respond to guiding questions as posted on the CANVAS course web site. You are required to make at least one original posting and respond to at least two group member postings per week.
3. Be knowledgeable of all the assigned readings, problem tasks, videos, and RUanalytic work.
4. Create a VMCAnalytic using the RUanaltyic tool from the video collection housed on the *Video Mosaic* (videomosaic.org) showing growth in student reasoning and the conditions contributing to that growth (This satisfies a portfolio requirement for those students matriculated in the mathematics-education, MEd, program).
5. Complete a *reflection paper* to include your learning through online discussions about videos, problem solving, readings, project work, collaborative work with peers, and your knowledge of the mathematics, research on how students learn, and implications for teaching with regard to NCTM and Common Core Standards. You may review your postings on the course web site and notes from problem solving and sharing of solutions as you develop your reflective assessment.

Description of Activities- See Course Outline below for schedule and requirements. Other readings and video/VMCAnalytic viewings will be assigned throughout the course. Modifications in assignments such as readings and video/analytic study will be made as judgment suggests.

Class sessions are held online. **Three additional synchronous, group problem-solving sessions will be held as new tasks are introduced using ZOOM. These special sessions will be announced, as needed, for students who would like to work collaboratively on tasks**

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and/or obtain assistance with their course project. Attendance at these sessions, while voluntary, is strongly recommended.

A FEW WORDS ON "NETIQUETE" - This is drawn from Palloff, R. M., & Pratt, K. (1999). *Building learning communities in cyberspace*. San Francisco: Jossey-Bass, p. 101.

1. Check the discussion frequently and respond appropriately and on the subject.
2. Focus on one subject per message and use pertinent, informative, and not-too-long titles
3. Capitalize words only to highlight a point or for titles; otherwise viewed as SHOUTING.
4. Be professional, respectful, and careful with your online interaction
5. Cite all quotes, references, and sources.
6. When posting a long message, it is generally considered courteous to warn readers at the beginning of the message that is a lengthy post.
7. It is inappropriate to forward someone else's message(s) without their permission.
8. Use humor carefully. The absence of face-to-face cues can cause humor to be misinterpreted as criticism or flaming (angry, antagonistic criticism). Feel free to use emoticons such as :-) or ;-) to let others know that you're being humorous.

COURSE SCHEDULE

The following outline is a tentative course schedule. Please refer to the CANVAS website weekly for exact assignments. From time to time, we may find it helpful to schedule some synchronous virtual interactions either through a conference call or webinar (e.g., ZOOM). As need or interest suggests, these will be scheduled.

WEEK	ACTIVITIES	READINGS
Wk 1 **9/1 – 9/7**	**I. Canvas Assignments** Complete your background information. Submit to **Canvas Assignments.** Title background **BEGINNING with your name.** **II. Download** & review: K-8 Algebra Common Core State Standards Initiative (2010).	**I. Readings for Week 1** 1. Erlwanger, S. H. (1973). 2. Maher, C. A. & Weber, K. (2010) **II. Online discussion per guiding questions about the readings**
Wk 2 **9/8-9/14**	**I. Canvas Assignments** Solve **Geese, Ladders**, and **Museum** problems and submit to Canvas Assignments; save file beginning with your name. **II. Study the following VMCAnalytics** 1. Kayla Albrethsen: James' Recognition of the Isomorphism Between the Museum Problem and	**I. Readings for Week 2:** 1. Wilkinson, L. C. (2019). Learning language & mathematics: A perspective from linguistics & education. 2. Sigley & Wilkinson (2016).

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	the Ladder Problem http://dx.doi.org/doi:10.7282/t3-b6j1-4r22 2. Ariel Constructing Linear Equations for "Guess My Rule" and the "Ladder" Problems http://dx.doi.org/doi:10.7282/T3NG4SD7 3. Tracing Ariel's Algebraic Problem Solving: A Case Study of Cognitive and Language Growth https://doi.org/doi:10.7282/T3N0186C	3. Moschkovitch (2018). Talking to learn mathematics with understanding. **II. Online discussion per guiding questions about the reading** **III. Discuss what is meant by "algebraic reasoning".**
Wk 3 **9/15-9/21**	**I. Study the following VMCAnalytics:** 1. An Experiential Investigation of Algebraic Ideas Created and Implemented by Robert B. Davis http://dx.doi.org/doi:10.7282/T3NV9G6H 2. Robert B. Davis Engages Students in Finding the Sum and Product Rule for Quadratic Equations https://doi.org/doi:10.7282/T3HQ41QB 3. Davis introduces students to "Guess My Rule" Activities https://rucore.libraries.rutgers.edu/rutgers-lib/46857/emap/1/standalone 4. Notice Davis' teaching style https://rucore.libraries.rutgers.edu/rutgers-lib/47181/emap/1/standalone	**I. Readings for Week 3:** 1. Davis, R. B. (1992). Understanding understanding 2. Davis, R. B. & Maher, C. A. (1990). 3. Maher, C. A. & Davis, R. B. (1995). Children's explorations leading to proof. In C. Hoyles and L. Healy (eds.), *Justifying and proving in school mathematics* **II. Online discussion per guiding questions about the reading**
Wk 4 **9/22-9/28**	**I. This week is devoted to individual and partner problem solving, in preparation for studying how students reason and build justifications to problem tasks.** **II. Canvas Assignments Solutions** **Submit convincing arguments to justifying the number of:** (a) Towers, 3-tall, selecting from 3 colors (b) Towers, 5-tall, selecting from 2 colors	**I. Readings for Week 4** Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.) 1. Chapter 1: *The Longitudinal Study* 2. Chapter 2: Methodology **II. Online discussion per guiding questions about readings**

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	(c) Pizza, 4-topping, Submit a convincing argument for finding the total number of possible pizzas, selecting from 4 toppings III. Study videos: Stephanie/Dana (Grades 2 and 3) http://dx.doi.org/doi:10.7282/T3MC8Z77 http://dx.doi.org/doi:10.7282/T3R49Q0D	
Wk 5 **9/29-10/5**	**Students' Problem Solving in the Counting Strand: Examining Students' Strategies and Solutions** **I. Study the following VMCAnalytics** 1. Stephanie, Dana, Jeff and Milin: Gang of 4 (Gr 4) http://dx.doi.org/doi:10.7282/T3CC0ZND 2. PUP-Math Pizzas (Gr 5) Parts 1 and 2 http://dx.doi.org/doi:10.7282/T3HM57PQ http://dx.doi.org/doi:10.7282/T3NC60FW 3. Inductive Reasoning (Gr5) http://dx.doi.org/doi:10.7282/T39C707G **II. Schedule Zoom meeting with partner or small group to share and discuss problem solutions.**	**I. Readings for Week 5** Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.) 1. Chapter 4: Towers, Schemes, Strategies, Arguments 2. Chapter 5: Building an Inductive Argument 3. Chapter 6: Making Pizzas: Reasoning by Cases and by Recursion 4. Maher, C. A., & Martino, A. M. (1996) **II. Online discussion per guiding questions about readings and video narratives**
Wk 6 **10/6-10/12**	**Students' Problem Solving in the Counting Strand: Studying students 'reasoning and use of representations** **I. Study the following VMCAnalytics:** 1. PUP-Math Brandon Interview http://dx.doi.org/doi:10.7282/T3VX0FRD 2. Brandon's Aha http://dx.doi.org/doi:10.7282/T3VH5R01 **II. Canvas Assignments Solutions**	**I. Readings for Week 6** 1. Maher, C. A. & Martino, A. (1998) 2. Greer, B., & Harel, G. (1998) 3. Skemp, R. R. (1976) **II. Online discussion per guiding questions about readings and videos/analytics**

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	Submit convincing arguments to justifying the solution to Ankur's Challenge.	
Wk 7 **10/13-10/19**	**Students' Problem Solving in the Counting Strand: Attending to Problem Structure** **I. Study the following VMCAnalytics:** 1. Romina's Proof to Ankur's Challenge (Gr. 10) http://dx.doi.org/doi:10.7282/T30P0Z85 2. Stephanie's Algebraic Solution (Gr 8) http://dx.doi.org/doi:10.7282/T3FN180C 3. Stephanie's Geometric Reasoning http://dx.doi.org/doi:10.7282/T3QZ2CRF **II. Schedule Zoom meeting with partner or small group to share and discuss problem solutions.**	**I. Readings for Week 7** 1. Maher, C. A. (2009). Children's Reasoning: Discovering the Idea of Mathematical Proof Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.) 2. Chapter 8: Ankur's Challenge **II. Online discussion per guiding questions about readings, video, and analytic**
Wk 8 **10/20-10/26**	**Students' Problem Solving Counting Strand: Attending to Structure and Building Connections** **Study the following VMCAnalytics:** 1. PUP-Math Night Session http://dx.doi.org/doi:10.7282/T34F1Q0W 2. PUP-Math, Stephanie (Gr 8) Pascal's Addition http://dx.doi.org/doi:10.7282/T3862FPR 3. Pascal's Identity (Grade 11) http://dx.doi.org/doi:10.7282/T3VX0FMM 4. Developing Isomorphic Relationships (Grade 11) http://dx.doi.org/doi:10.7282/T3H1310N	**I. Readings for Week 8** 1. Maher, C. A. (2005). Insights from a long-term study. Combinatorics and Reasoning book (Maher, Powell & Uptegrove, Eds.): 2. Chapter 12: Representations and Standard Notation **II. Online discussion per guiding questions about readings, video, and analytic**
Wk 9 **10/27-11/2**	**Fraction Strand** **I. Studying how to create a VMCAnalytic and how to produce a first draft. Review Tutorial:** Making a VMCAnalytic (with RUanalytic tool)	**I. Readings for Week 9:** 1. Chapter 1: Maher & Yankelewitz (2017). Children's Reasoning While Building Fraction Ideas

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	II. Obtaining, as needed assistance in creating VMCAnalytic and learning to use RUanalytic tool **III. Study the following video clip: Introduction to Informal Math Learning Project (IML): Establishing Norms with Sixth Graders via Rods Lesson:** https://drive.google.com/file/d/1ZuMfdYdli787JxfkO0H96dknhcnOaPqg/view **IV. Synchronous Session, to be scheduled, to discuss rod models and creating VMCAnalytic**	2. Van Ness & Maher (2017). Analysis of the argumentation of nine-year-olds engaged in discourse about comparing fraction models **II. Draft of Analytic Outline. Be sure to include the transcript and time codes for the Video Events.**
Wk 10 **11/3-11/9**	**Children's Reasoning Fraction Strand** **I. Study the following VMCAnalytics:** 1. Establishing Norms and Creating a Mathematical Community http://dx.doi.org/doi:10.7282/T30C4XH9 2. Task Design Prompts Fourth Grade Students http://dx.doi.org/doi:10.7282/T3ZK5JF0 3. Fourth Graders Reason by Cases as They Explore Fraction Ideas http://dx.doi.org/doi:10.7282/T3Q2420N 4. An Introduction to Comparing Unit Fractions http://dx.doi.org/doi:10.7282/T3V4010R	**I. Readings for Week 10** 1. Chapter 2: Establishing a Mathematical Community, Gerstein 2. Chapter 3: A Problem with No Solution, Yankelewitz & Winter 3. Chapter 5: Reasoning by Cases while Exploring Fractions as Numbers, Winter & Yankelewitz 4. Chapter 7: Establishing the Importance of the Unit, Van Ness & Alston **II. Online discussion per guiding questions about readings, video, and analytic** **III. First draft of your VMCAnalytic to be shared with a partner:** Partner with at least one other student to share and receive feedback about your VMCAnalytic; also, partner with at least one other student to provide feedback on their VMCAnalytic. Please note that the file title should begin with your NAME.
Wk 11 **11/10-**	**I. Study the Fraction Strand VMCAnalytics:**	**I. Readings for Week 11**

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11/16	1. Comparing 1/2 and 1/3: Confusion about the Unit http://dx.doi.org/doi:10.7282/T3ZW1NS9 2. Fourth Graders' Analyses of Equivalence: 1/5 or 2/10 http://dx.doi.org/doi:10.7282/T3WW7KFN 3. Development of Upper and Lower Bound Arguments Comparing Fractions http://dx.doi.org/doi:10.7282/T3ZS2Z8N	1.Chapter 8: Switching the Unit, Alston & Van Ness 2. Chapter 10: Determining Fractional Equivalence, Gerstein & Yankelewitz 3. Chapter 12. The Development of Upper and Lower Bounds Arguments while Comparing Fractions, Yankelewitz & Winter **II. Online discussion per guiding questions about readings, video, and analytic** **III. Feedback to/from partner's VMCAnalytic project**
Wk 12 **11/17-11/23**	**I. Study the Fraction Strand VMCAnalytics:** 1. Imagining the Density of Fractions http://dx.doi.org/doi:10.7282/T3FJ2JKN 2. Using Meredith's Models to Reason About Comparing and Ordering Unit Fractions http://dx.doi.org/doi:10.7282/T33J3FQG 3. Extending Fraction Placements from Segments to Number Line: Obstacles and Their Resolutions http://dx.doi.org/doi:10.7282/T39Z96SR	**I. Readings for Week 12** 1. Chapter 15: From Rod Models to Line Segments, Schmeelk 2. Chapter 17: Comparing and Ordering Fractions, Horwitz & Schmeelk 3. Chapter 18: Extending Fraction Placement from Segments to Number Line, Horwitz **II. Online discussion per guiding questions about readings and analytics** **III. PROJECT:** Review and revise your VMCAnalytic as appropriate from feedback received
Wk 13 **11/24-11/29**	**Thanksgiving Recess**	**No class scheduled**

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Wk 14 **11/30-12/7**	**I.** Reporting/Reviewing **VMCAnalytic Projects** in online discussion **II.** Begin **Reflection paper**	**Submit VMCAnalytic:** Instructors must **approve** VMCAnalytic text before putting text online.
Wk 15 **12/8-12/13**	**Exit Survey**	**Canvas Assignments** **1. FINAL** VMCAnalytic Project due **2. Reflection** paper due

Required Text (Available as paper back and online version – rental also available)

Maher, C. A. & Yankelewitz, D. (2017). *Children's Reasoning While Building Fraction Ideas*. Sense Publishers.

https://www.springer.com/us/book/9789463510080

Recommended Text (rental also available)

Maher, C. A., Powell, A. B. & Uptegrove, E. (Eds.), (2010). *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* New York: Springer Publishers.

https://www.springer.com/us/book/9789400706149
https://www.amazon.com/Combinatorics-Reasoning-Representing-Isomorphisms-Mathematics-ebook-dp-B008BC0LS0/dp/B008BC0LS0/ref=mt_kindle?_encoding=UTF8&me=&qid=

Supplemental Readings

Agnew, G., Mills, C. M., & Maher, C. A. (2010). VMCAnalytic: Developing a collaborative video analysis tool for education faculty and practicing educators. In R. H. Sprague, Jr. (Ed.), *Proceedings of the 43rd Annual Hawaii International Conference on System Sciences (HICCS-43): Abstracts and CD-ROM of Full Papers*. IEEE Computer Society, Conference Publishing Services: Los Alamitos, CA.

Ball, D. L. & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, G. W. Martin, and D. Schifter, (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational psychologist*, *31*(3-4), 175-190.

Common Core State Standards Initiative. (2010). Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Davis, R. B. (1992). Understanding 'understanding' (1992). *The Journal of Mathematical Behavior*, *11*, 225-241

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Davis, R. B., & Maher, C. A. (1990). Chapter 5: What do when we do mathematics"? Journal for Research in Mathematics Education. Monograph, 4, 65-210.

Erlwanger, S. H. (1973). Benny's Conception of Rules and Answers in IPI Mathematics. *The Journal of Children's Mathematical Behavior* 1(2), 7-26.

Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior*, *24*(3), 361-372.

Greer, B., & Harel, G. (1998). The role of isomorphisms in mathematical cognition. *The Journal of Mathematical Behavior*, *17*(1), 5-24.

Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. *Mathematical Cognition*, 175-225.

Lampert, M. & Cobb, P. (2003). Communication and language. In J. Kilpatrick, G. W. Martin, and D. Schifter, (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 227-249). Reston, VA: National Council of Teachers of Mathematics.

Maher, C. A. (2005). How students structure their investigations and learn mathematics: Insights from a long-term study. *The Journal of Mathematical Behavior*, *24*(1), 1-14.

Maher, C. A. (2009). Children's reasoning: Discovering the idea of mathematical proof. In M. Blanton, D. Stylianou and E. Knuth (Eds.), *Teaching and learning proof across the K-16 curriculum* (pp. 120-132). New Jersey: Taylor Francis - Routledge.

Maher, C. A. & Ahluwalia, A. (2014). Counting as a foundation for learning to reason about probability. In E. J. Chernoff & B. Sriraman (Eds.), *Probabilistic Thinking: Presenting Plural Perspectives* (pp. 559-580). Springer: New York, NY.

Maher, C. A. & Martino, A. (1998). "Brandon's Proof and Isomorphism". In C. A. Maher, *Can teachers help children make convincing arguments? A glimpse into the process.* Rio de Janeiro, Brazil: Universidade Santa Ursula.

Maher, C. A., & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*, 194-214.

Maher, C. A., Powell, A. B. & Uptegrove, E. (Eds.), (2010). *Combinatorics and reasoning: Representing, justifying and building isomorphisms.* Springer Publishers

Maher, C. A. & Speiser, R. (1997). How far can you go with block towers? Stephanie's Intellectual Development. *The Journal of Mathematical Behavior, 16*(2), 125-132.

Maher, C. A. & Weber, K. (2010). Representation Systems and Constructing Conceptual Understanding. Special Issue of the *Mediterranean Journal for Research in Mathematics Education 9*(1), 91-106.

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Moschkovich, J. (2018). Talking to learn mathematics with understanding: Supporting academic literacy in mathematics for English learners. In *Language, Literacy, and Learning in the STEM Disciplines* (pp. 13-34). Routledge.

Pedemonte, B. (2007). How can the relationship between argumentation and proof be analysed? *Educational Studies in Mathematics*, 66(1), 23-41

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